# THEORIE ELEMENTAIRE DU CERF-VOLANT ELEMENTAL KITE THEORY by Emile BERTINET, 1887 

## Contribution to the reading and the understanding of this book, by C. Becot

## PROLOGUE

This book of sixty-seven pages is sometimes cited in French bibliographies on kite. It is almost ignored in English language bibliographies. When cited, there is no comment or description of the work. Its content is really unknown as well as the first name of its author, whose initial is only seen.
The purpose of this prologue is to introduce Émile Bertinet and his work. Then, a reading guide explain and to get acquainted with it.
A few points, not covered by Bertinet but which we know today, are also added.

## A double publication

It's a rare book. Fortunately, it is possible to consult it and make a copy at the documentation centre of the Musée de l'Air et de l'Espace at Le Bourget in France.
On the internet, search engines do not find any copies online. However, in 2012 I realized that Bertinet had been a teacher at the Reims high school, was a member of the Academy of Reims, and that his book was published as "Librairie de l'Académie". Knowing the annual editions of the National Academy of Reims, I consulted them on Gallica, the internet publication of the National Library of France. First observation, the printer is the same: F. Michaud, in Reims. I open volume 81, volume 1 of the years $1886-1887$, and I see that Bertinet's work is published in its entirety!
The only difference is the numbering of the pages since in the volume of the academy of Reims it is with many other articles. Here is the link to access this document :
http://gallica.bnf.fr/ark:/12148/bpt6k5746678p/f86.image.langFR

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pu
CERF-VOLANT

Par E. bertinet




hbims


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The book itself

TRAVAUX

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Issue 81 volume 1 of the Annales

Bertinet was a member of the National Academy of Reims from 1885 to 1888 only during his professorship at the Lycée de Reims. The activity report of the Secretary General H. Jadart tells us that at the time of this publication Bertinet had already begun to present his work on the theory of the kite, and also that he made observations on the flight of birds at the carillon tower of Notre-Dame de Reims.
In another volume I noticed an article by Bertinet entitled Causerie sur la navigation aérienne, published after his reading on June 7, 1888. Bertinet would then go on to teach in Paris, first at the Lycée Charlemagne, later at the Lycée Buffon.
He then lived at 2 rue Michelet in Issy-Les-Moulineaux.
The publication on "Le vol des oiseaux" that he mentions in his work at the end of the section "Bases of Theory" will be made at the Academy of Sciences and published in volume 105 of the second half of 1887 .
He will also write a dissertation "Contribution au vol des oiseaux "which he will present to the Academy of Sciences in 1889.

## A work of hard reading

Not only has this book remained inaccessible for a long time, but it is really hard to read for two main reasons.

First of all, there is no table of contents. The chapters are not numbered; there are no page breaks between them. The typography of the sections, paragraphs, sub-paragraphs does not help to identify them correctly. In mathematical analyses, the different cases follow one another in a row without distinction. The 32 figures are grouped together in two plates, extremely nested within each other, which saves one or two pages to engrave, but makes their numbers difficult to find and their visualization complicated.
Its presentation therefore makes it difficult to approach and to understand this book.
The second reason is that a good part of the book is a series of demonstrations and transformations of mathematical formulas. Thus, one quickly loses the thread of reasoning. In addition, a solid mathematical background is necessary because some of the concepts are at university level and preparatory classes for engineering schools.
It is difficult to distinguish which are the final formulas; the definitions of variables are scattered throughout the book, which makes it difficult to investigate when it is necessary to go back over part of the text.

So I suspect that the few people who had the curiosity to open this book quickly closed it. It also clearly explains the lack of information or quotations on the content of this atypical work.

The great particularity of Bertinet is to start from very simple physical observations and laws, and by the sole force of an ingenious mathematical handling to lead to the determination of quantities and characteristic equilibria.
What is even more remarkable lies in the practical conclusions he draws from them.
With the following reading guide I will now try to extract the substantial marrow from it. Everything that is presentation and explanation is in black. Formulas and equations are in bistre. My additional comments to the subject are in green.

You will be able to understand the concept of the theory and the balance of the kite even without mathematical knowledge.
This reading guide alone is a complete summary to approach the Theory of the Kite, whether you are a scientist or not.

## READING GUIDE

## Content page

After going through this book, my first concern was to reconstruct the table of contents. Herebelow is the list of chapters and the number of pages devoted to them :

Description du cerf-volant
Bases de la théorie
Cerf-volant ordinaire
Application
Cerf-volant muni d'une surcharge
Cerf-volant cas général
Discussion (et résumé)
Autres modes d'attache de la ficelle
Forme à donner au cerf-volant
Utilité du cerf-volant

1 page Kite description
2 pages Basis of the theory
18 pages Ordinary kite
2 pages Application
6 pages Kite with overload
7 pages General case of the kite
17 pages Discussion (and summary)
3 pages Other ways to tie the line
2 pages Shape to give to the kite
2 pages Utility of the kite

This way you can see the plan of the work and the importance of each part. However, if one removes the tedious mathematical demonstrations and retains only the concepts and principles, the book becomes of great interest. The table of contents is presented in detail in the appendix with the paginations of both the separate edition and the publication of the Annales de l'Académie de Reims.
Let us now try to go through it, chapter by chapter, using the names of the physical quantities used by Bertinet.

## Description of the kite

A short and easy chapter that reflects the thoughts and concerns of the time.
Bertinet made many observations from a diamond-shaped kite with a tail.
One cannot help but think of Mr Esterlin's one described in Nature in 1887, taken up by A. Batut, who flies without a tail.

## Bases of the theory

The aerodynamics of the kite is derived from the wind pressure on a rectangular surface of dimensions $a \times b$ and inclined at an angle $\alpha$. See Figure 1.
The wind, of speed $w$, exerts a thrust $N^{\prime}=N \sin \alpha=a \cdot b \cdot 0,1 \cdot w^{2} . \sin \alpha$ knowing that on the kite in flight the angle $\alpha$ varies with the wind speed.
The application point of the thrust force is at the centre when the surface is perpendicular to the wind and moves away from it towards the leading edge at distance of a/6. $\cos \alpha$

This is the usual formula $\mathrm{N}=\mathrm{kS} \mathrm{V}{ }^{2}$ with S the surface and V the wind speed. The value of 0.1 of the coefficient is, according to other authors, between 0.07 and 0.13 depending on the shape of surfaces, their size, etc... These values are probably optimistic applied to kites. As Bertinet uses a lot k for another definition, I will replace by convention the value 0.1 by u /10 in the formulas to remember that this value needs to be better determined.


Fig. 1


## Ordinary kite

The sections in this chapter show the structure of reasoning and demonstration:
$1^{\circ}$ Shape, weight, pressure, center of thrust.
$2^{\circ}$ Mode of operation
$3^{\circ}$ Equilibrium position - balance stability
$4^{\circ}$ Values of $\phi$ lift force at departure
$5^{\circ}$ Height reached; descent
$6^{\circ}$ Wire tension


## $1^{\circ}$ Shape, weight, pressure, center of thrust.

Bertinet describes it as follows: width $\mathrm{AB}=2 \mathrm{a}$ and height $\mathrm{CD}=3 \mathrm{a}$.
The intersection $O$ is at the distance $C O=a$. The surface area is $3 a^{2}$.
He calculates the weight from the surface and its density d, i.e. $3 a^{2} \delta$.
It determines the position of the centre of gravity G which is below O at $\mathrm{OG}=\mathrm{a} / 3$.
Note that the density is what today we call the wing loading.
It confirms the thrust at $90^{\circ}$ and deduces the thrust at angle $\alpha: N \alpha=u / 10.3 . a^{2} \sin \alpha . W^{2}$ It determines the position of the centre of thrust at the distance $(a \cos \alpha) / 3$ above G .
To do this, he reasons on a thin rectangular band mm'nn', see fig. 3 , and he makes the integration for the entire surface of the kite; it is a simple mathematical problem.

## $2^{\circ}$ Mode of operation

This section should be read step by step, guided by the text. Indeed, Bertinet comments how the physical quantities interact on the kite simply held vertically on its horizontal axis AB. It starts with the inclination $\alpha$ at takeoff, then the flight until reaching the maximum elevation.
fig 4 front wind

fig 5 inclined for take-off


For each wind speed there is a corresponding inclination $\alpha$. During the ascent the kite remains parallel to itself. $\phi$ is the upward force. The kite stabilizes when the resultant of the thrust N and the weight P is aligned with the kite line angle.

## $3^{\circ}$ Equilibrium position - balance stability

First of all Bertinet tries to determine at what angle the kite will maintain itself for a given wind speed w . To do this he applies the equations of equilibrium of the moments of the forces acting on the kite, that is to say : the thrust of the wind and the weight. Then he juggles with mathematics to determine the angle of inclination $\alpha$.
He shows that there are two solutions for which the kite can find its balance. One is $\alpha$ between $0^{\circ}$ and $90^{\circ}$ and the second is $\alpha$ between $180^{\circ}$ and $270^{\circ}$.
Bertinet then proposes a graphical method to determine the angle of inclination by drawing two curves. Each curve corresponds to a part of the equation.

The solutions, i.e. the values of $\alpha$ are the points of intersection of these two curves:

$$
\begin{aligned}
& y^{\prime}=\cos \alpha \\
& y^{\prime \prime}=k \sin \alpha \cdot(1-\cos \alpha) \quad \text { with } \quad k=u / 10 \cdot w^{2} / \delta
\end{aligned}
$$

The $y^{\prime}$ curve is simple. The $y^{\prime \prime}$ curve is more complex. For those who know how to use them, calculators and computer programs can be used to plot them. Bertinet, on the other hand, had to look for singular points and calculate the slope of the $y^{\prime \prime}$ curve at these points in order to be able to draw its shape. In the end he shows that for the inclination $\alpha$ between $0^{\circ}$ and $90^{\circ}$ the equilibrium is stable while it is unstable for the solution of $\alpha$ between $180^{\circ}$ and $270^{\circ}$. Obviously, the value between $0^{\circ}$ and $90^{\circ}$ is the only one we observe.

## $4^{\circ}$ Values of $\phi$; Lift force at departure

Bertinet now calculates the lift force $\phi$ at departure.

$$
\phi=s \cdot \delta \cdot(k \sin \alpha \cdot \cos \alpha-1)
$$

Then he applies the equation of the angle of inclination $\alpha$ and he gets $\cos \alpha=0,618$
Which is $\alpha=52^{\circ}$. This angle seems to be the maximum rearing angle of the kite but it is absolutely not explicit.
Let's take a kite of $1 \mathrm{~m}^{2}$ and wing loading $\mathrm{d}=0.25$. It thus weighs 0.25 kgf . With $52^{\circ}$ we obtain a rising force of 0.9 kg at the start, 0.6 kg with an incidence of $22^{\circ}$ and 0.18 kg with an incidence of $10^{\circ}$. These values are plausible.
For the kite to stand in the air, continuing his calculations, Bertinet came up with this formula:

$$
0,1 \cdot w^{2}>2,058 \delta \quad \text { or also } \quad w^{2}>20,58 \delta
$$

Is the minimum speed calculated by this formula realistic? Let's assume a wing loading of 0.25 . This formula gives $2.3 \mathrm{~m} / \mathrm{s}$. The reality is a wind speed of more than $3 \mathrm{~m} / \mathrm{s}$.

The most realistic formula that I know is: minimum wind speed $>12 \delta$

## $5^{\circ}$ Height reached; Descent

With the forces and their directions in fig. 9 Bertinet writes their equation of equilibrium, where $\beta$ is the angle of the wire with the horizontal.


Bertinet then comes up with the formula that determines this wire angle :

$$
\operatorname{Tg} \beta=\left[\cos ^{2} \alpha+\cos \alpha-1\right] /[\sin \alpha \cdot \cos \alpha]
$$

Taking $\alpha=22^{\circ} \delta=0,25$ a wind of $5 \mathrm{~m} / \mathrm{s}$ and $\mathrm{u} / 10=0,1$ we get $\mathrm{k}=10$ giving $\beta=66^{\circ}$ which is quite realistic for a maximum wire angle
With other considerations, Bertinet shows that if the kite raises more than $51^{\circ}$ the aircraft will descend. He therefore explains that depending on the wind, and for a given length of line, the kite moves between two altitudes, the lower given by the maximum angle of incidence (pitching up) and the upper with the minimum angle of incidence.

## Case in point:

With $\delta=0.25$ and a minimum wind speed of $3 \mathrm{~m} / \mathrm{s}$ we get $\mathrm{k}=3.6$ and $\beta=3^{\circ}$, i.e. the kite remains practically at ground level. It needs more wind to rise

## $6^{\circ}$ Wire tension

Bertinet reminds the tension at the start (kite raiser) : $\mathrm{T}=\mathrm{u} / 10 . \mathrm{w}^{2} . \mathrm{s} \cdot \sin \alpha$
And in another position of inclination: $\quad T^{\prime}=s . \delta \cdot[k \cdot \sin \alpha \cdot \sin (\alpha+\beta)-\sin \beta]$
Taking the values $\alpha=22^{\circ} \delta=0.25$ at a wind speed of $5 \mathrm{~m} / \mathrm{s}$ and $\mathrm{u} / 10=0.1$ for a $1 \mathrm{~m}^{2}$ kite we obtain a tension of 1.9 kg at take-off with an angle of $51^{\circ}$ and 0.7 kg in flight with an angle of incidence $\alpha=22^{\circ}$ and an angle of flight $\beta$ of $66^{\circ}$. These results are plausible.
Bertinet then establishes a general formula for the yarn tension, which depends solely on the wing loading and the angle of incidence :

$$
\mathrm{T}^{2}=\mathrm{s}^{2} \delta^{2}\left[2 \cos ^{3} \alpha-2 \cos \alpha+1\right] /[1-\cos \alpha]^{2}
$$

With the above values, we find a tension of 3.2 kg . This value is obviously strong and different from the value calculated from the above $\mathrm{T}^{\prime}$ formula. When $\alpha$ decreases, T increases which is abnormal. Is there a typographical error? In my experiments, under the same conditions, the measured tension is $\sim 1 \mathrm{~kg}$ for a bearing surface of $1 \mathrm{~m}^{2}$.

This section ends with a table of values based on this formula for angles $\alpha$ in $5^{\circ}$. Negative values are of no interest. How to use this table is actually explained in the next chapter.
The sum $\alpha+\beta$ is always less than $90^{\circ}$. The ratio $(\alpha+\beta) / 90^{\circ}$ gives the efficiency of the kite. We can see that it increases when the incidence decreases.
However the sum never reaches $90^{\circ}$ as it is the case in this table for values below $15^{\circ}$.

## Application

This table is made for only one type of kite: the one in fig.3.
First the $k$-value has to be calculated as a function of the wind speed $w$ and the wing loading $\delta$ of the kite. Having $k$, one reads from the table, where one interpolates the angle of incidence $\alpha$ and the angle of flight $\beta$. In the same way the ratios $\phi / \mathrm{p}$ and $\mathrm{T} / \mathrm{p}$ are determined from the table. P being the weight of the kite it is sufficient to multiply the ratios by the weight of the kite to determine the actual values of the uplift force $\phi$ and the tension on the wire T .
I have tabulated the values of k as a function of wind speed w and wing loading $\delta$ for an aerodynamic coefficient $\mathrm{u} / 10=0.1$.


It can be seen that the values of $k$ are limited, here from 0.1 to 20 , for already extreme wind conditions (2 to 8 Beaufort) and wing loading. Applying these values to Bertinet's table shows that there is an inconsistency; k being limited, it would not be possible to have an incidence of less than $25^{\circ}$. I therefore wonder about the purely theoretical aspect or about the fact that some relevant hypotheses would not have been taken into account.

All in all, in these two chapters on the ordinary kite in fig. 3 there is a simple and implacable methodology to rationally determine essential data on the flight of this kite.
However, this kite is unable to fly properly without tail; therefore, some results are approximate and cannot be absolute results. This deficiency will be discussed further.

## Kite with overload

At first, Bertinet considers the tail as a simple dead weight that shifts the center of gravity. This leads him to the following question: What position should the center of gravity be in order for the aircraft to climb as high as possible?
He then examines how the flight angle $\beta$ varies when the position of the center of gravity $G$ moves. The normal centre of gravity is at the distance $a / 3$ below the transverse axis $A B$. Bertinet expresses the variable position of G as m a/3 with m varying from 1 to 0 between the axis $A B$ and $G$ and $>1$ beyond.
He thus arrives at the formula: $\quad m=2\left[1-k / \sqrt{ }\left(4+k^{2}\right)\right]$
With $\mathrm{k}=10$ which is an incident angle at $32^{\circ} \mathrm{m}=0.039$ i.e. a displacement $\sim 4 \%$. When the incident angle decreases towards zero, $m$ tends towards 1 .

## Tension

Bertinet then researches the tension to be exerted to hold a kite in place. The answer comes quickly : this effort is independent of the strength of the wind, and is equal to the weight of the kite. He writes in conclusion: this result is very curious.
When $\alpha=45^{\circ}$ then $\mathrm{T}=\mathrm{p}$ see fig. 12. Actually, there is a mistake somewhere.


## Conclusions

$1^{\circ}$ By making the kite heavier or moving the center of gravity away, the kite will fly lower.
$2^{\circ}$ If the center of gravity is close to $A B$, a light tail will have a beneficial effect, especially for low k values when the kite is struggling to climb.
$3^{\circ}$ By lowering the center of gravity, the tail has a considerable influence on stability.
Indeed, the greater the moment induced, the greater the inertia. Thus, the greater the distance, the greater the moment, the greater the inertia, and the more the kite can withstand the effects of the wind. Otherwise, there is over-reaction. If the oscillation produced is too great, there is a reversal of the phenomena, and the kite can lose its lift and fall.
With a weak moment, the kite will not be able to rearing again because the resistance of the air during its fall will prevent it, and "the kite will fall to the ground".

## Remarque

Bertinet thus thinks to calculate the weight that a kite can take and determine where best to place it on the kite.

Bertinet's approach is relevant. Many of his remarks are consistent with reality. Clearly, he lacked concrete data to apply his formulas. Some of them do not correspond to what we can observe and measure at present.
He was probably missing an essential instrument : the anemometer.

## General case of the kite

The structure of this chapter is exactly the same as the ordinary kite chapter.

The reasoning followed by Bertinet is therefore strictly the same.
$A$ and $B$ are the bridle points.
Initially, the kite pivots on this line.


## $1^{\circ}$ Shape, weight, thrust, center ot thrust.

This first section is self-explanatory, and you only need to read it as Bertinet wrote it, but remember to make two breaks in the text, one for weight, the other for pressure.
Not to forget: the aerodynamic force $N=0,1 . W^{2}$. s. $\sin \alpha$
its position, at the distance $(a . \operatorname{Cos} \alpha) / 3$ above the center of gravity with the half-height of the kite $a$, and its area s.

Density, specific weight, wing loading are the same physical quantity. As defined by Bertinet, the word density is perfectly understandable and appropriate.

## $2^{\circ}$ Mode of operation

Again, this section should be read step by step, guided by the text.

## $3^{\circ}$ Positions of equilibrium

In this section everything is fine the first two lines, until we turn the page.
Quickly we run to see fig. 15

$S$ is the intersection between $A B$ and CD (fig14).
H is the centre of thrust. The thrust is $\mathrm{N}^{\prime}$.
$G$ is the centre of gravity, the point of application of the weight $P$.
After rotation around $A B$, when the resultant $T$ of the thrust and weight passes to point $S$, the kite takes off. This is when Bertinet explains Fig. 15.
To construct the resultant, start from the intersection I of $N^{\prime}$ and $P$. Plot the length of $N^{\prime}$; from the end, plot $P$. The resultant goes from intersection I to the latter end.

Bertinet calculates the distance SH which depends on the angle of incidence a since H varies with a. Recall that O being the geometric center of the rectangle, m expresses SH as a fraction of the distance $\mathrm{a}=\mathrm{OC}=\mathrm{OD}$ and r expresses the distance OG as a fraction of a, i.e. $\mathrm{SH}=\mathrm{m}$ a and $\mathrm{OG}=\mathrm{ra}$
It sounds complex but facilitates mathematical manipulation.
Then he writes the equations of moments, N ' with distance SH and P with distance GH .
$(m-\rho) \cos \alpha-k \cdot \sin \alpha[m-(\cos \alpha) / 3]=0 \quad \# 1$ (equation $n^{\circ} 1$ )
Using $k$ that we know well, he arrives after mathematical manipulations to a formula containing only $\mathrm{a}, \mathrm{k}, \mathrm{m}$ and r but with tangents power 4 of $\alpha .(\operatorname{tg} \alpha)^{4}$

## $4^{\circ}$ Lift force at departure, ascensionnelle au départ; elevation reached ; tension

Bertinet gives the lift force, s being the sail area :

$$
\phi=s \cdot \delta \cdot[k \cdot \sin \alpha \cdot \cos \alpha-1] \quad \# 2 .
$$

He does the calculation of the line angle $\beta$ with the formula:

$$
\operatorname{tg} \beta=[k \cdot \sin \alpha \cdot \cos \alpha-1] /\left[k \sin ^{2} \alpha\right] \quad \# 3
$$

From that, the elevation of the kite is calculated by :
$H=L . \sin \beta$ where $L$ is the length of the line.
He ends this chapter with the formula of the tension of the line:

$$
\mathrm{T}=\mathrm{s} \cdot \delta[\mathrm{k} \cdot \operatorname{sina} \cdot \sin (\alpha+\beta)-\sin \beta] \quad \# 4
$$

This chapter ends with a two-point conclusion, which I reproduce in full here:
$1^{\circ}$ When the kite has taken a position of equilibrium around AB and is subjected to a force $\phi$, it goes up or down without destroying its equilibrium; $\phi$ decreases in a continuous way, from its value at the beginning to zero.
$2^{\circ}$ The tension of the line gradually increases from the kite's starting position to its equilibrium position in the air. $\phi$ and T for equal values of k and $\alpha$ are proportional to the surface area of the kite.
In this chapter, Bertinet writes: When the line will do with the horizon an angle $\beta, \phi$ will have the value: $\phi=s . \delta \cdot[k \cdot \sin \alpha \cdot \cos (\alpha+\beta)-\cos \beta]$

I must confess that I do not understand this formula, which differs from the previous one. The upward force being strictly vertical, and the thrust depending only on the incidence, there is no reason to see here the angle of the thread, especially since $\alpha$ varies with $\beta$.

## Discussion

We thought we'd already understood and learned everything, but we haven't!
$E$. Bertinet explains everything once more!
This chapter begins by recalling equations \#1 to \#4 above as well as the formula for the coefficient $\mathrm{k}=0,1 . \mathrm{w}^{2} / \delta$ and the formula for the area $\mathrm{s}=2 \mathrm{a} \mathrm{b}$.

Bertinet first develops that since the equation of moments \#1 depends neither on the width nor the length, but on the position of the center of gravity (by m) and the bridle (by r), kites with these two settings in the same proportions will fly at the same height with the same line length. In addition, kites of the same density, or wing loading d, will have the same behavior in winds of the same speed.

Then he analyzes the different values that $\mathrm{m}, \mathrm{r}$ and k can take in the equation of moments. He calls the support axis the direction of the line angle passing through the bridle point.
If you have a lot of patience, if you draw and follow the diagram of solutions and cascade choices carefully, as you go through these 15 pages, you will see that most of the time, either there is no possible solution, or the kite falls. Sometimes there is a realistic solution. You will then have the pleasure to understand all the cases of figures 16 to 26 .

The simplest and most rational way is to go to the summary with its four parts I to IV.
There are two new equations that are \#5 and \# 6 :
The one that allows us to know the maximum wire angle: $\operatorname{tg} \beta=\left(k^{2}-4\right) / 4 \mathrm{k} \quad \# 5$
And this one that I reformulated as follows: $\rho+m=2 / 3 k / \sqrt{ }\left(k^{2}+4\right) \quad \# 6$
Knowing k and the position of the centre of gravity, it enables the position of the clamping axis to be calculated for a wire angle b , and thus for the theoretical maximum.

Since, as the angle of incidence the wire angle is related to the coefficient k , I therefore calculated with equation \#5 the wire angle $\beta$ according to values of k between 0.2 and 20 (corresponding to the range of values previously defined as plausible) depending on the wind speed and wing loading of the kites.

| angle de fil | $\operatorname{tg} b=\left(k^{2}-4\right) / 4 \mathrm{k}$ |  |
| :---: | :---: | :---: |
| $\mathbf{k}$ | tangente | angle |
| 0,2 | $-5,0$ | $-79^{\circ}$ |
| 1 | $-0,8$ | $-37^{\circ}$ |
| 1,5 | $-0,3$ | $-16^{\circ}$ |
| 2 | 0,0 | $0^{\circ}$ |
| 3 | 0,4 | $23^{\circ}$ |
| 4 | 0,8 | $37^{\circ}$ |
| 5 | 1,1 | $46^{\circ}$ |
| 7 | 1,6 | $58^{\circ}$ |
| 9 | 2,1 | $65^{\circ}$ |
| 12 | 2,9 | $71^{\circ}$ |
| 16 | 3,9 | $76^{\circ}$ |
| 20 | 5,0 | $79^{\circ}$ |

For $k<2$ the angle $\beta$ is negative.
For $k=2, \beta$ is zero, and $\beta$ is positive for $k>2$.
I wondered before why Bertinet often arrived at the condition $\mathrm{k}>2$ for the kite to fly, it is now obvious.
The result of the calculation also shows that for $5<k<20$ the line angle varies between $45^{\circ}$ and $80^{\circ}$, range of satisfactory flying conditions.

Actually, I find this k coefficient very interesting

## Other ways to tie the line

This chapter is divided into two distinct parts I and II. It is really worth reading it entirely.

## Part I

Fig. 27 By attaching the kite at two symmetrical points on the line $A B$, or even at a single point in the center, the kite has identical behavior and stability because the CDS plane remains vertical.


If the $A B C D$ plane of the kite is not perpendicular to the wind, but oblique, it naturally returns to the equilibrium position which is perpendicular to the wind.
$1^{\text {st }}$ Remark


If the CDS plane is no longer vertical, it is the weight of the kite that brings it back to its point of equilibrium.
This reflection of Bertinet shall be noted.
$2^{\text {nd }}$ Remark
With a kite of any surface, but symmetrical, we are returned to the same method as before.
The weight of the string is not taken into account, but its influence is negligible.

In reality the action of the string does not affect the behavior of the kite as long as the tension is greater than its weight. If the tension is lower, the kite falls.

Furthermore, there is a phenomenon that we know well :
When the incidence is low and the tension has decreased significantly, the weight of the string will add to the equation of moments and prevent the kite from pitching up to return to its equilibrium position.

## Partie II



Bertinet deals with two-point bridle on the vertical axis CD of the kite. He explains how to find the optimum bridle point. First the angle of incidence and the desired angle of flight must be determined; then the position of M1 must be calculated, i.e. the value of $m$ in the formula \#1 or \#6. From M1 draw an angle $\alpha+\beta$. All that remains to be done is to draw OM and ON and to determine their lengths. M must be far enough forward so that the center of thrust is below.

## Shape to give to the kite

In fact, it is not the search for the most suitable form that Bertinet offers us, but the optimisation of a given form. And for this exercise, he chose the rectangle.

He calculates the position of the support axis $A B$ from equation \#6. Thus $\mathrm{m}=2 / 3$.
Considering that $\mathrm{a}+\mathrm{b}$ is close to $90^{\circ}$ when the kite is flying high, it is easy to position the bridle.

He then gives a numerical example.


## Utility of the kite

I let you discover and enjoy the Bertinet's life-saving kite proposal, close to the current stunt kites, and its solution to fix the mooring point of the line to the ground.
It was a very fashionable subject. I do not know if Bertinet experimented with his system.
Bertinet probably did not imagine that on the seashore there is either sand or rocks. In the sand the pick won't hold, and in the rocks it won't penetrate. For the same weight, a light but strong hook would be more likely to be used as an anchor.

## Epilogue

This work by Emile Bertinet deserves to be known. His theoretical approach is undeniably correct. It remains to be adjusted with data from experimentation.
T. Bois in 1905 will do so in his book "Les cerfs-volants et leurs applications militaires", The kites and their military applications.

## E. BERTINET <br> THÉORIE ÉLÉMENTAIRE DU CERF-VOLANT

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